

**Civil Systems Investment Planning and Pricing (12-706) /  
Quantitative Methods for Policy Analysis (19-702) /  
Economic Analysis of Private and Public Projects (73-359)**

**(Individual) Homework 2 (Due Monday Sept. 19 - 10% penalty per day late)**

**Question 1 (15 pts): Note: This is similar to an example I showed in class. However I expect you to solve this problem using the discounting equations rather than Excel.**

Assume that it is your 22<sup>nd</sup> birthday (for some of you that is in the future – but that is not important). You decide to start saving for retirement which will start at age 66. You are trying to decide between 2 savings plans:

*Plan 1:* You invest \$2,000 per year (starting today) for the next  $n$  years on your birthday. You then leave the money alone earning interest, and withdraw the entire balance on your 66<sup>th</sup> birthday. So you make a total of  $n$  payments.

*Plan 2:* You wait  $n$  years before starting to invest the \$2,000 per year, but then you continue to invest the money every year until you are 66 (and then withdraw all of the money). Thus you make a total of  $44-n$  payments.

- a) [10 pts] Assuming an 8% effective annual interest rate for both plans, find  $n$  so that the plans have approximately the same amount available at your 66<sup>th</sup> birthday.
- b) [5 pts] What is  $n$  if you assume continuous discounting?

**Question 2 (20 pts):** A benefit-cost study of a proposed dam is conducted. The dam costs \$75 million to construct. The study estimates a continuous stream of social benefits of \$9.5 million per year (from avoided flood damage, hydroelectric power, etc.) and costs of \$4 million (\$2 million from operation and \$2 million in environmental damages).

**Note: Feel free to use Excel/etc.** on this problem.

- a) [8 pts] Assuming a social discount rate of 4% per year, **how many years** does it take for the dam to “break even” (i.e., the NPV of benefits just exceed the NPV of costs).
- b) [12 pts] Opponents of the study disagree with using a single discount rate of 4% for all of the benefits and costs. They argue that the \$2 million in environmental costs should only be discounted at 1%. **Develop a graph** of the NPV as a function of the years of operation – from 0 to 300 years – for this group of opponents. Assume all benefits and costs are \$0 after the end of operation of the dam. **What range of years of operation** would they be in favor of the dam?

**Question 3** (15 pts): Many of you have received financial aid through student loans to pay for your education. In the US, there are 2 types of student loans – subsidized and unsubsidized. In unsubsidized loans, students borrow money directly from banks at a certain (high) interest rate. In subsidized loans, students borrow money from the government (who is borrowing from banks but effectively pays part of the interest and gives students a lower interest rate).

- a) [5 pts] Assuming the unsubsidized rate is 8% and the subsidized rate is 3%. If you borrow \$20,000 and sign a contract to repay the loan in 10 years, how much would your monthly payments be for each type of loan?
- b) [5 pts] For both loans, how much total interest do you pay? Compare the total interest with the total amount repaid for both loans.
- c) [5 pts] If your personal discount rate/MARR is 12%, how much do you currently value the government loan subsidy?

**Question 4** (10 pts): Generally, a firm has two options if it wants a new piece of equipment. First, it could **purchase** the equipment itself, allowing it to deduct depreciation and interest expenses when calculating taxes, as discussed in class. It could also **lease** equipment, but then only the leasing expenses are deductible for tax purposes (since they do not own it, they can not depreciate the asset). In the terms used in class, the after-tax cash flow at time t from leasing is as follows:

$$A_t = B_t - C_t$$

$$Y_t = A_t - A_t X_t = A_t - T_t$$

Your company is considering the acquisition of a computer system for \$22,000. You may either buy the computer with borrowed money at 8%, with a single repayment at the end of 5 years, or lease it with yearly payments of \$7500, \$6500, \$5500, \$4500, and \$3500 at the end of years 1 through 5. The salvage value is \$2000, and the expected yearly benefits are \$6000. Your corporate income tax rate is 50%, depreciation is by double declining balance, and discount rate is 8%.

- a) [7 pts] Should you buy or lease?
- b) [3 pts] Does your answer change if your discount rate is 5%? What if it is 15%